L'Électrostriction Intrinsèque et Sa Perte de Densité Énergétique

The Intrinsic Electrostriction and its Energy Density Loss

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Résumé

Les matériaux électrostrictifs, qui présentent une déformation quadratiquement liée au champ électrique appliqué, offrent un potentiel considérable pour une utilisation dans les actionneurs, les capteurs et d'autres applications. Ce comportement contraste avec celui observé dans les matériaux piézoélectriques, où la déformation est directement proportionnelle au champ électrique. Dans les diélectriques électrostrictifs pratiques et non idéaux, une hystérésis est observable à la fois dans la polarisation en fonction du champ électrique, démontrant une non-linéarité, et dans la déformation en fonction du champ électrique, qui s'écarte d'une forme parabolique en raison d'un déphasage par rapport au champ électrique. Un important corpus de recherche a été entrepris pour calculer et quantifier les pertes d'énergie associées à ces écarts, comme en témoigne l'élargissement de la boucle d'hystérésis polarisation-champ électrique. De manière analogue, il est anticipé que des pertes d'énergie se produiront pendant le processus d'électrostriction, en raison de l'existence d'une « ouverture » similaire dans la boucle déformation-champ électrique. Néanmoins, les mécanismes intrinsèques de l'électrostriction et les pertes d'énergie associées n'ont pas été suffisamment étudiés. Cette étude propose un cadre pour élucider l'électrostriction intrinsèque et quantifier ses pertes d'énergie. Globalement, elle fournit une quantification complète des différentes formes d'énergie — d'entrée, de perte et d'énergie stockée — dans les processus diélectriques et électrostrictifs intrinsèques.

Mots Clés : Électrostriction ; Couplage électromécanique ; Coefficient de couplage complexe

Abstract

Electrostrictive materials, which exhibit a strain that is quadratically related to the applied electric field, offer considerable potential for utilisation in actuators, sensors, and other applications. This behaviour is in contrast to that observed in piezoelectric materials, where the strain is directly proportional to the electric field. In practical, non-ideal electrostrictive dielectrics, hysteresis is observable in both the polarization as a function of the electric field, demonstrating nonlinearity, and the strain as a function of the electric field, which deviates from a parabolic form due to a phase lag relative to the electric field. A substantial body of research has been undertaken to calculate and quantify the energy losses associated with these deviations, as evidenced by the widening of the polarization-electric field hysteresis loop. Analogously, it is anticipated that energy losses will occur during the electrostriction process, due to the existence of an analogous 'opening' in the strain-electric field loop. Nevertheless, the intrinsic mechanisms of electrostriction and the associated energy losses have not been subjected to sufficient investigation. This study proposes a framework for elucidating the intrinsic electrostriction and quantifying its energy losses. Overall, it provides a comprehensive quantification of the different forms of energy—input, loss, and stored energy—in both the dielectric and intrinsic electrostrictive processes.

Keywords : Electroctrostriction,Electromechanical coupling ; complex coupling coefficient

1. Introduction

Tab. 1. – Comparison of Piezoelectricity and Electrostriction

Electro-mechanical Effect	Expression Formula	Requirements
Piezoelectricity	$x_{ij} = d_{ijk}E_k$	only in non-centrosymmetric dielectrics
Electrostriction	$x_{ij} = M_{ijkl}E_kE_l$	in all dielectrics

Table 1 provides a comparative analysis of piezoelectricity and electrostriction. Piezoelectricity, expressed as $x_{ij} = d_{ijk}E_k$, is proportional to the electric field but only exists in non-centrosymmetric dielectrics[1]. In contrast, electrostriction, expressed as $x_{ij} = M_{ijkl}E_kE_l$, exhibits a quadratic dependence on the electric field and is universally present in all dielectrics regardless of symmetry [2]. This fundamental difference makes electrostriction more versatile and broadly applicable, significant potential.

Electrostriction represents an electro-mechanical coupling phenomenon in which electrical energy (input) is converted into mechanical energy (output). Energy efficiency is a critical parameter for practical applications of electrostriction, such as in actuators. The energy conversion efficiency can be described as $\eta = \frac{\text{output}}{\text{input}}$. In practical scenarios, the output energy is invariably smaller than the input energy, indicating the presence of energy losses inherent to the electromechanical coupling process [3].

There are at least 2 loss mechanisms in the context of electrostriction : dielectric loss and mechanical loss [4]. Under ideal circumstances, if there is no other loss mechanism, the input energy minus output energy should equate the summation of dielectric loss and mechanical loss. If the combined losses from these two mechanisms are found to be less than the observed discrepancy between input and output energies, it implies the existence of additional, unidentified loss mechanisms.

Kenji Uchino, in his seminal work, identified various loss mechanisms that can arise in piezoelectric materials when subjected to external electric fields [4]. While these electromechanical losses in piezoelectric systems have been extensively characterized in the literature, notably by Uchino and others, significantly fewer studies have focused on the analogous dissipation phenomena in electrostrictive materials. In particular, the mechanisms of energy loss associated with electrostriction-induced strain under high electric fields remain poorly understood and are seldom addressed in existing research. Although the study presented in [5] introduces a modeling approach for electrostriction losses, it is primarily based on low-field and low-stress conditions. Yet, when electrostrictive materials are subjected to strong electric fields alone—without external mechanical loading—they still undergo macroscopic strain, and the associated energy dissipation processes remain unclear. What types of losses emerge in this purely field-driven regime ? How can they be quantitatively assessed ? Beyond dielectric and mechanical losses, are there additional loss channels involved ? The present work seeks to address these questions by systematically investigating the loss mechanisms active during electrostriction under high electric fields.

This study seeks to identify and characterize these potential loss mechanisms and provide a precise quantitative expression for each contributing factor to the overall energy dissipation. Extensive research has shown that dielectric loss is associated with the imaginary component of dielectric permittivity ε or susceptibility χ, and mechanical loss relates to the imaginary component of the dynamic elastic modulus G. Dielectric loss equates the area enclosed by the polarization-electric field loop and the mechanical loss is the area enclosed by the strain-stress loop. Similarly, in this work, we find that the electrostrictive coefficient is inherently complex q. Amplitude represents capability of conversion from electric to mechanical energy, while the imaginary component (or phase angle) leads to energy losses, described as the "intrinsic electrostriction loss." It is important to note, however, that intrinsic electrostriction loss is not simply defined by the area enclosed by the strain-electric field loop. This distinction arises due to the inherently quadratic relationship between strain and the electric field. A deeper understanding of this nonlinear behavior is crucial for quantifying this intrinsic electrostriction loss.

2. Theory part

2.1. Prerequisite knowledge

2.1.1. The Hilbert Transform

The Hilbert transform is transforming a time-dependent signal $s(t)$ into an analytical signal $\tilde{s}(t)$ composed of its complex amplitude $|s|$, its angular frequency ω and its phase φ, where $s(t) = \mathfrak{Re}\{\tilde{s}(t)\}$.

For instance, a sinusoidal signal $s(t)$ is described as :

$$s(t) = |s| \sin(\omega t + \varphi)$$

the analytical signal $\tilde{s}(t)$ corresponding to the real signal $s(t)$ is obtained from the following process :

$$\begin{aligned}\widetilde{s}(t) &= s(t) + i \cdot \mathcal{H}[s(t)] \\ &= |s| \cos(\omega t - \pi/2 + \varphi_s) + i \cdot |s| \sin(\omega t - \pi/2 + \varphi_s) \\ &= |s| e^{i(\omega t - \frac{\pi}{2} + \varphi_s)}\end{aligned}$$

where $\mathcal{H}[\cdot]$ is the Hilbert transform.

2.1.2. Notation rules

The notations adopted hereafter are the following : complex quantities are denoted with a tilde so that : $Z = \mathfrak{Re}\{\tilde{Z}\}$ **for any real quantity** Z. The Hilbert transform is not applicable to square quantities, *e.g.*, for the electric field E : $E^2 = \mathfrak{Re}\{\widetilde{E^2}\} \neq \mathfrak{Re}\{(\tilde{E})^2\}$. The following notations are therefore adopted, for a sinusoidal electric field :

$$E(t) = |E| \sin(\omega t) = |E| \cos\left(\omega t - \frac{\pi}{2}\right) \quad \text{(Eq. 1a)}$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency and T its period.The square of $E(t)$ is :

$$E^2 = |E|^2 \sin^2(\omega t) = \frac{|E|^2}{2} [1 + \cos(2\omega t - \pi)] \quad \text{(Eq. 1b)}$$

and the complex electric field is :

$$\widetilde{E} = |E| e^{i(\omega t - \pi/2)} \text{ with } \mathfrak{Re}\{\widetilde{E}\} = |E| \cos\left(\omega t - \frac{\pi}{2}\right) \quad \text{(Eq. 1c)}$$

and its square is :

$$(\widetilde{E})^2 = |E|^2 e^{i(2\omega t - \pi)} \quad \text{(Eq. 1d)}$$

whereas the complex form of the squared electric field is :

$$\widetilde{E^2} \equiv \frac{|E|^2}{2} \left(1 + e^{i(2\omega t - \pi)}\right) \quad \text{(Eq. 1e)}$$

The same notations will be used for the polarization P.

2.2. Ideal dielectrics

The curve of polarization as a function of the electric field for ideal dielectrics is :

$$P(E)_{\mathrm{ID}} = \sum_{n=1,2,3,...} \varepsilon_0^n \chi^{(n)} E^n \qquad \text{(Eq. 2)}$$

where ε_0 is vacuum permittivity and $\chi^{(n)}$ is n-th order electric susceptibility, then the time revolution of polarization is, by substituting Eq. 1a into Eq. 2 :

$$P(t)_{\mathrm{ID}} = \sum_{n=1}^{\infty} \varepsilon_0^n \chi^{(n)} |E|^n \sin^n(\omega t) \qquad \text{(Eq. 3)}$$

the curve of strain as a function of the electric field is :

$$x(E)_{\mathrm{ID}} = \sum_{n=1,2,3,...} M^{(n)} E^n \qquad \text{(Eq. 4)}$$

where $M^{(n)}$ is the n-th order electromechanical coupling coefficient, $M^{(1)}$ is piezocoefficient, and $M^{(2)}$ is the electrostrictive coefficient, where $\chi \in \mathbb{R}$, $M^{(n)} \in \mathbb{R}$.

2.3. Ideal liner electrostrictors

For ideal linear electrostrictors ($\chi^{(n)} = 0$ for $n \geq 2$, and $M^{(n)} = \delta_{n,2}$), the polarisation is proportional to electric field and strain is quadratic in the electric field (parabola) :

$$P_{\mathrm{ILE}} = \varepsilon_0 \chi E = \varepsilon_0 \chi |E| \sin(\omega t) \qquad \text{(Eq. 5)}$$

$$x_{\mathrm{ILE}} = M^{(2)} E^2 = M^{(2)} |E|^2 \sin^2(\omega t) \qquad \text{(Eq. 6)}$$

2.4. Non-ideal dielectrics

$$P(t)_{\mathrm{NID}} = \sum_{n=1,2,3,...} \varepsilon_0^n |\chi^{(n)}| |E|^n \sin^n(\omega t - \varphi_{P_n}) \qquad \text{(Eq. 7)}$$

Using the following transformation,

— The power-reduction formula of trigonometric identities
— A phase shift normalization $-k * \frac{\pi}{2}$
— Combine like terms
— Hilbert transform
— Discrete Fourier transform
— Define discrete Fourier weighs
— Real part

Then $P(t)_{\mathrm{NID}}$ is related to the Discrete Fourier Series :

$$P(t)_{\mathrm{NID}} = |\tilde{P}_0| + \left\lfloor \frac{2}{N} \right\rfloor \sum_{k=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left| \widetilde{F_P}(k\omega) \right| \cos(k\omega t - \varphi_{\tilde{F}_{P_k}}) \qquad \text{(Eq. 8)}$$

Similarly, the time evolution of the strain $x(t)_{\mathrm{NID}}$, which is associated with the Discrete Fourier Series, can be represented as :

$$x(t)_{\mathrm{NID}} = \sum_{n=1,2,3,...} M^{(n)} |E|^n \sin^n(\omega t - \varphi_{x_n}) \qquad \text{(Eq. 9)}$$

$$x(t)_{\mathrm{NID}} = |\tilde{x_0}| + \left\lfloor \frac{2}{N} \right\rfloor \sum_{k=1}^{\left\lfloor \frac{N}{2} \right\rfloor} \left| \widetilde{F_x}(k\omega) \right| \cos(k\omega t - \varphi_{F_{x_k}}) \qquad \text{(Eq. 10)}$$

2.5. Non-ideal linear electrostrictors

The time evolution of strain for non-ideal linear electrostrictors, denoted as $x(t)_{\text{NILE}}$, associated with Discrete Fourier series, is given by :

$$x(t)_{\text{NILE}} = M^{(2)}|E|^2 sin^2(\omega t - \varphi_{x_2}) = \frac{1}{2}M^{(2)}|E|^2 + \frac{1}{2}M^{(2)}|E|^2 \cos(2\omega t - \pi - 2\varphi_{x_2}) \qquad \text{(Eq. 11)}$$

$$\tilde{x}(t)_{\text{NILE}} = x_0' + \left\lfloor \frac{2}{N} \right\rfloor \left|\widetilde{F}_x(2\omega)\right| e^{i(2\omega t - \varphi_{F_{x_2}})} = x_0' + \left\lfloor \frac{2}{N} \right\rfloor \widetilde{F}_x(2\omega) e^{i(2\omega t)} \qquad \text{(Eq. 12)}$$

2.6. The intrinsic electrostriction and its energy density loss

The input energy density (the electrical energy density) is equal to the sum of the following components : the output energy density, mechanical energy density loss, dielectric energy density loss, and intrinsic electrostriction energy density loss.

2.6.1. The dielectric process

The recoverable energy density in the dielectric process is :

$$u_{\text{rec}}^{\text{diel}} = \varepsilon_0|\chi||E|^2 \cos\varphi_{P_1} \qquad \text{(Eq. 13)}$$

The energy density loss in the dielectric process is [4] :

$$u_{\text{loss}}^{\text{diel}} = \pi\varepsilon_0|\chi||E|^2 \sin\varphi_{P_1} \qquad \text{(Eq. 14)}$$

The input energy is the summation of the recoverable energy density and energy density loss :

$$u_{\text{input}}^{\text{diel}} = u_{\text{rec}}^{\text{diel}} + u_{\text{loss}}^{\text{diel}} = \varepsilon_0|\chi||E|^2(\cos\varphi_{P_1} + \pi\sin\varphi_{P_1}) \qquad \text{(Eq. 15)}$$

2.6.2. The mechanical process

The mechanical loss is :

$$\begin{aligned} u_{\text{loss}}^{\text{Mech}} = 2\pi &\left\{\left\lfloor \frac{2}{N} \right\rfloor |F_{\text{total}}(2\omega)|\right\}^2 |G| \sin\varphi_G + \\ &\frac{\pi}{2}\left\lfloor \frac{2}{N} \right\rfloor |F_{\text{total}}(2\omega)|\varepsilon_0|\varepsilon_r||E|^2 \sin(-\varphi_{\varepsilon_r} + \varphi_{F_{x_2}} + 2\varphi_G) + \\ &\frac{\pi}{2}\left\lfloor \frac{2}{N} \right\rfloor |F_{\text{total}}(2\omega)|\varepsilon_0|\varepsilon_r||E|^2 \sin(\varphi_{\varepsilon_r} - \varphi_{F_{x_2}}) + \\ &\frac{\pi\varepsilon_0^2|\varepsilon_r|^2|E|^4}{8|G|} sin\varphi_G \end{aligned} \qquad \text{(Eq. 16)}$$

the output energy in the whole process equates the recoverable energy in the mechanical process, The input energy in the mechanical process,denoted as $u_{\text{input}}^{\text{Mech}}$ is :

$$u_{\text{input}}^{\text{Mech}} = \frac{\pi\sin\varphi_G + \cos\varphi_G}{\pi\sin\varphi_G} u_{\text{loss}}^{\text{Mech}} \qquad \text{(Eq. 17)}$$

The input energy density minus the loss energy density in the mechanical process is the output energy in the whole process.

2.6.3. The intrinsic electrostriction and its energy density loss

Flowchart

$$\begin{array}{ccc} \tilde{E} & \xrightarrow{\tilde{P}=\varepsilon_0\tilde{\chi}\tilde{E}} & \tilde{P} \\ \tilde{x}^{\mathrm{ES}}=\frac{1}{2}(|M^{(2)}|-\widetilde{M^{\mathrm{Maxwell}}})|E|^2+\frac{1}{2}\widetilde{M^{\mathrm{ES}}}(\tilde{E})^2)\downarrow & & \downarrow \tilde{X}^{\mathrm{ES}}=\tilde{q}\tilde{P} \\ \tilde{x}^{\mathrm{ES}} & \xleftarrow[\tilde{x}^{\mathrm{ES}}=\frac{\tilde{X}^{\mathrm{ES}}}{\tilde{G}}]{} & \tilde{X} \end{array}$$

The transformation from an applied electric signal to the resulting mechanical strain can be structurally represented as the energy transfer pathway shown in the above Flowchart. This process involves multiple coupled physical mechanisms—dielectric polarization, Maxwell-induced stress, and mechanical deformation—each contributing distinct recoverable and loss energy components.

This model provides a comprehensive framework for decomposing the total input electric energy into physically meaningful terms, each associated with specific stages of the electromechanical conversion process. It reveals that, in non-ideal linear electrostrictors, energy dissipation arises not only from conventional dielectric and mechanical losses, but also from the intermediate coupling process—most notably, the transformation from polarization to stress.

By resolving the Maxwell-induced contribution and quantifying its recoverable and dissipative parts, the model enables the isolation of a previously unaccounted loss component : the intrinsic electrostriction loss. This loss arises from the irreversible energy consumed during the polarization-to-stress conversion and is otherwise concealed within overlapping mechanisms.

Importantly, this formulation not only clarifies energy allocation at each conversion stage, but also introduces a new energy-based descriptor for evaluating electrostrictive performance across different materials under harmonic excitation. It extends beyond classical dielectric loss or mechanical damping metrics, offering a complete frequency-domain perspective on energy dissipation.

From the viewpoint of total energy flow, the electric input energy is partially stored and dissipated through multiple processes : the dielectric response, mechanical response, and Maxwell-induced stress.

Each process has been quantitatively decomposed into recoverable and loss terms. However, the sum of all these resolved contributions remains insufficient to fully account for the input electric energy.

The energy flow transferred from the electric excitation to mechanical strain can be expressed as follows :

$$u_{\mathrm{electric}}^{\mathrm{input}}+u_{\mathrm{input}}^{\mathrm{Maxwell}}=u_{\mathrm{reco}}^{\mathrm{mech}}+u^{\mathrm{dielectric\ loss}}+u^{\mathrm{mechanical\ loss}}+u^{\mathrm{intrinsic\ ES\ loss}}+u_{\mathrm{loss}}^{\mathrm{Maxwell}}+u_{\mathrm{reco}}^{\mathrm{Maxwell}} \quad \text{(Eq. 18)}$$

Due to the relation $u_{\mathrm{input}}^{\mathrm{Maxwell}}=u_{\mathrm{loss}}^{\mathrm{Maxwell}}+u_{\mathrm{reco}}^{\mathrm{Maxwell}}$, then the intrinsic electrostriction loss can be quantified using the following formula :

$$u_{\mathrm{electric}}^{\mathrm{input}}=u_{\mathrm{reco}}^{\mathrm{mech}}+u^{\mathrm{dielectric\ loss}}+u^{\mathrm{mechanical\ loss}}+u^{\mathrm{intrinsic\ ES\ loss}} \quad \text{(Eq. 19)}$$

3. Conclusion and perspective

This work establishes a comprehensive frequency-domain framework for analyzing dielectric and electrostrictive behaviors under harmonic electric excitation. The discussion begins by defining the complex dielectric susceptibility using Hilbert transform, enabling time-domain signals to be interpreted in terms of their complex magnitude and phase. This complex representation is then extended to electromechanical coupling coefficients via discrete Fourier transform, allowing harmonic terms in the strain and polarization response to be quantitatively related to their corresponding material parameters.

Building upon this formalism, we introduced the concept of energy density sequences, where the energy flow is traced from input electric energy density to the recoverable mechanical strain energy density. The energy pathway is decomposed into three physical processes : the dielectric process, the mechanical process, and the Maxwell stress-induced process. For each, we derived the respective input energy density, recoverable energy density, and associated loss energy density, grounded in physical conservation principles.

To validate the framework, we applied this analysis to two representative materials : the inorganic relaxor PMN-10PT and the soft polymer PDMS. In both cases, we experimentally extracted the energy densities involved in each subprocess. However, we found that the sum of all known loss and recoverable energies—across dielectric, mechanical, and Maxwell branches—falls short of the total input electric energy. This discrepancy indicates the presence of an additional, previously unaccounted loss mechanism.

Through detailed interpretation of the electromechanical energy conversion chain, we identified this missing component as the energy loss occurring during the transformation from polarization to mechanical stress. We define this as the *intrinsic electrostriction energy density loss*—a new physical quantity that captures energy dissipation intrinsic to the polarization-to-stress conversion.

Notably, this intrinsic loss was found to be material-dependent. For the PDMS sample, the intrinsic electrostriction loss was negligible, whereas for PMN-10PT, it accounted for a significant portion of the energy dissipation. This suggests that intrinsic electrostriction loss is a nontrivial and variable contributor to the total energy conversion efficiency.

The introduction of this loss component adds a new layer of physical insight to the study of electromechanical systems. By separating the intrinsic electrostriction loss from dielectric and mechanical losses, this framework provides a new direction for analyzing and optimizing energy conversion devices. Future work can build on this foundation to minimize all three types of loss—dielectric, mechanical, and intrinsic electrostriction—through targeted material design and control of phase lag parameters, ultimately enhancing the energy efficiency of electroactive systems.

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