Optimisation des descripteurs des champs de topologie et d'anisotropie pour les problèmes de conduction thermique

Optimisation of the topology and anisotropy fields descriptors for thermal conduction problems

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Résumé

Ce travail présente une nouvelle méthode d'optimisation simultanée de la topologie et de l'anisotropie de structures composites à rigidité variable abriquées avec des technologies de additive manufacturing. L'approche proposée utilise une stratégie d'optimisation à plusieurs niveaux basée sur des entités NURBS, une méthode d'optimisation de la topologie basée sur la pseudo-densité et le formalisme polaire pour décrire l'anisotropie du matériau. Les entités NURBS sont utilisées pour représenter le champ de densité et la distribution des paramètres polaires dans le domaine de conception. L'optimisation vise à maximiser la conductivité thermique de la structure tout en répondant aux exigences de conception en matière de légèreté et de faisabilité des paramètres polaires. L'efficacité de la méthodologie est évaluée à l'aide de problèmes de référence tirés de la littérature, pour lesquels l'optimisation est étendue au cas d'optimisation simultanée de la topologie et de l'anisotropie.

Abstract

This work presents a new method for the simultaneous optimisation of topology and anisotropy field descriptors for variable stiffness anisotropic structures issued by additive manufacturing. The proposed approach uses a multi-level optimisation strategy based on NURBS entities, a density-based topology optimisation method and the polar formalism to describe the anisotropy of the material. NURBS entities are used to model the density field and polar parameter distribution in the design domain. The optimisation aims to maximise thermal conductivity while meeting design requirements for lightness and polar parameter feasibility. The effectiveness of the methodology is evaluated using benchmark problems from the literature, where the optimisation is extended to the case of simultaneous optimisation of topology and anisotropy.

Mots Clés : Optimisation Topologique, Conducitivité Thermique, Anisotropie, NURBS, Parametres Polaires.
Keywords : Topology optimisation, Heat conduction, Anisotropy, NURBS, Polar Parameters.

1. Introduction

Since the advent of additive manufacturing, mechanical designers face challenges in determining the optimal topology and material composition of a continuum. These objectives are typically addressed by considering structural design at the macroscopic scale and material design at the mesoscopic scale. The primary goal is to find the optimal distribution of a given material in the design domain to maximize structural performances while respecting some constraints. A classical problem often addressed in the literature is the one of the maximisation of the structural stiffness subject to a constraint on the volume of the solid phase composing the structure. Typically, a topology optimisation (TO)

method is used for this purpose, where the merit function used to evaluate stiffness is related to the work of the external forces. Secondly, the optimisation of material properties is considered. In this case, it is possible to determine the optimal material properties by taking into account the already established optimal topology. However, this method may lead to suboptimal solutions when compared to the concurrent optimisation of both the topology and the anisotropy, as shown in previous studies [1]. Indeed, anisotropic materials, such as composites, can often present better performance than their isotropic counterparts, depending on the specific design requirements of the problem. Two different types of multilayer composite structures are used in industrial applications : those with constant stiffness, which are characterised by a straight fibre path, and the variable stiffness composites (VSCs), which are characterised by a curvilinear fibre path, and, possibly, variable thickness and variable volume fraction. Regarding the TO methods, the most used methods available in the literature are the level-set method [2] and density-based method [3]. As far as the design of VSC structures is concerned, the most efficient design method available in the literature is the multi-scale two-level optimisation strategy [4]. In this context, the design problem is split into two sub-problems stated at different scales (or levels). At the macroscopic scale the VSC laminate is considered as an equivalent single-layer plate with point-wise anisotropic behaviour described by a suitable representation of the laminate characteristic tensors describing stiffness, stress, thermal conductivity, etc. At this scale, the anisotropy of the VSC structure can be represented using lamination parameters (LP) combined with the Tsai and Pagano's representation [5] An alternative representation of the anisotropy is the Verchery's polar formalism [6], which provides a more flexible and physically meaningful approach compared to the LP. After determining the optimal distribution of the polar parameters (PPs) at the macroscopic scale, it is possible to state the problem at the mesoscopic scale where the goal is to find at least one optimal stacking sequence (and also the optimal fibres-path within each layer) matching the optimal distribution of PPs found at the macroscopic scale. Nevertheless, there is a lack of literature regarding the optimisation of thermal performances of VSC structures. The aim of this paper is to fill this gap and to propose a general design strategy to optimise concurrently the topology and anisotropy of VSC structures when considering heat conduction problems. Specifically, the problem is formulated in the framework of a density-based method based on Non Uniform Rational B-Spline (NURBS) entities [7] by proposing a general formulation of the optimisation problem based on the concept of generalised thermal compliance. In this background, a study on the influence of the penalisation schemes used to penalise the thermal conductivity matrix of the VSC laminate is conducted in this work. The effectiveness of the proposed method is tested on a meaningful 2D benchmark structure taken from literature [8].

2. Methodology

The polar formalism has been used for the design of VSC structures within the framework of the multi-scale two-level (MS2L) optimisation strategy, which is based on NURBS entities and introduced by Montemurro et al. [4]. This paper only considers the first level problem (FLP) at the macroscopic scale of the VSC structure, and this Section provides the formalism and fundamental features and tools necessary to properly state the problem when NURBS entities are used to describe both the topology and the anisotropy descriptors.

2.1. Topological field design variables

In this work the TO problem is solved using the NURBS-density-based method, developed at I2M laboratory in Bordeaux by Montemurro and coworkers [9]. This method has several interesting features that offer notable advantages. It uses a continuous NURBS entity of dimension $D + 1$, i.e., a purely geometric entity, to describe the topological descriptor, i.e., the pseudo-density field, for a problem of dimension D. This avoids dependence of the optimised solution to the finite element (FE) model mesh [9]. Additionally, the method is fully compatible with computer-aided design (CAD) software,

as the topology boundary is available in analytical form at each iteration, minimising post-processing operations to recover the solution boundary. Furthermore, the local support property of NURBS basis functions can reduce or eliminate the checkerboard effect, providing an efficient implicitly defined filter. Accordingly, for a 2D problem a 3D NURBS surface is used, whose third coordinate is the pseudo-density field, which reads :

$$\rho\left(\zeta_1, \zeta_2\right) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} R_{i,j}\left(\zeta_1, \zeta_2\right) \rho_{i,j}, \tag{Eq. 1}$$

where $R_{i,j}$ are the rational basis function of the NURBS entity [7] and dimensionless parameters ζ_j can be related to the spatial coordinates x_j as :

$$\zeta_j := \frac{x_j}{L_j},\ j = 1, 2, \tag{Eq. 2}$$

where L_j is the characteristic length of the domain along the x_j axis. The pseudo-density $\rho_{i,j}$ of the generic control point (CP) and the associated weight $w_{i,j}$ are considered as design variables, which are collected in the following vector :

$$\boldsymbol{\xi}_1^{\mathrm{T}} = \left(\rho_{i,j}, w_{i,j}\right)_{\substack{1 \le i \le n_1 \\ 1 \le j \le n_2}}, \boldsymbol{\xi}_1 \in \mathbb{R}^{2n_{\mathrm{CP}}}, \tag{Eq. 3}$$

where n_{CP} is the overall number of CPs of the NURBS entity.

2.2. Anisotropy field design variables

The VSC laminate is modelled as an equivalent homogeneous anisotropic single-layer plate at the macroscopic scale. The expression of any plane tensor of rank N by means of a set of tensor invariants is possible thanks to the Verchery's polar method [6]. It can be shown that the components L_{ij} (with $i, j = 1, 2$) of a second-rank symmetric tensor can be expressed as a function of the polar parameters (PPs) as :

$$\begin{aligned} L_{11} &= T + R\cos(2\Phi), \\ L_{12} &= R\sin(2\Phi), \\ L_{22} &= T - R\cos(2\Phi), \end{aligned} \tag{Eq. 4}$$

where T is the isotropic modulus, R is the deviatronic one and Φ is the polar angle. Among them, only T and R constitute the tensor invariants. For a second-rank symmetric tensor, the only possible symmetry is the isotropy that can be obtained when $R = 0$. In the framework of the linear thermal laminate theory (TLT) [10], the thermal local response of a quasi-homogeneous laminate , i.e., a laminate having equal in-plane and out-of-plane thermal conductivity matrices and with a null coupling conductivity matrix, is described by two PPs, i.e., $R^{\mathrm{K}^*_\theta}$ and $\Phi^{\mathrm{K}^*_\theta}$, which represent the anisotropic modulus and the polar angle of the homogenised in-plane thermal conductivity matrix $\mathbf{A}^*_\theta$. For optimisation purposes, we can consider the dimensionless quantities :

$$\alpha = \frac{R^{\mathbf{A}^*_\theta}}{R}, \qquad \phi = \frac{2\Phi^{\mathbf{A}^*_\theta}}{\pi}. \tag{Eq. 5}$$

where R is the deviatoric modulus of the thermal conductivity matrix of the constitutive ply. The design variables fields describing the behavior of the VSC laminate at the macroscopic scale are $\alpha\left(\zeta_1, \zeta_2\right)$ and $\phi\left(\zeta_1, \zeta_2\right)$. In contrast to the pseudo-density field ρ employed to represente the in-plane topology of the VSC laminate, the spatial variation of these variables is described using B-Spline entities, as no oscillations arise when optimising the PPs in the context of the MS2L optimisation approach [11, 12]. Accordingly, they read :

$$\nu\left(\zeta_1,\zeta_2\right)=\sum_{i=0}^{n_1}\sum_{j=0}^{n_2}N_{i,p_1}\left(\zeta_1\right)N_{j,p_2}\left(\zeta_2\right)\nu_{i,j},\qquad\text{with }\nu=\alpha,\phi, \tag{Eq. 6}$$

where $N_{i,p_k}\left(\zeta_k\right)$ are the Bernstein's polynomial defined along the k-th parametric coordinate of the B-spline entity [7]. In analogy with the topological design variables, the variables describing the anisotropy of the VSC structure at the macroscopic scale are α and ϕ computed at the CPs of the B-Spline entity. They are collected in the following vector :

$$\boldsymbol{\xi}_2^{\mathrm{T}}=\left(\alpha_{i,j},\phi_{i,j}\right)_{\substack{1\le i\le n_1\\1\le j\le n_2}},\quad \boldsymbol{\xi}_2\in\mathbb{R}^{2n_{\mathrm{CP}}}. \tag{Eq. 7}$$

2.3. Problem formulation

The optimisation strategy presented in this paper aims to determine the optimal distribution of field descriptors of both the anisotropy and the topology of the VSC structure to minimise the generalised thermal compliance C under a constraint imposed on the volume V of the structure. The design problem of the VSC structure is formulated as a constrained non linear programming problem (CNLPP) :

$$\min_{\xi_1,\xi_2}\frac{C(\xi_1,\xi_2)}{|C_{\mathrm{ref}}|}\text{ , subject to : }\begin{cases}\hat{\mathbf{K}}_\theta\hat{\theta}=\hat{\psi},\\ \dfrac{V}{V_{\mathrm{ref}}}\le\gamma,\\ \boldsymbol{\xi}_1\in[\rho_{\mathrm{LB}},\rho_{\mathrm{UB}}]\times[w_{\mathrm{LB}},w_{\mathrm{UB}}],\\ \boldsymbol{\xi}_2\in[0,1]\times[0,1]\times[-1,1].\end{cases} \tag{Eq. 8}$$

C_{ref} and V_{ref} are the reference values of the generalised thermal compliance and of the volume of the structure, respectively, whereas γ is the imposed volume fraction. $\rho_{\mathrm{LB}}=10^{-3}$ and $\rho_{\mathrm{UB}}=1$ represent the lower and upper bounds of the pseudo-density, $\omega_{\mathrm{LB}}=0.5$ and $\omega_{\mathrm{UB}}=10$ are the bounds for the weights and the ranges $[0,1]$ for α and $[-1,1]$ for ϕ represent the feasible domain for the dimensionless PPs. The problem is solved using a gradient descent algorithm, which requires an analytical formulation of the gradient of both thermal compliance and volume, whose analytical expression will be shown during the presentation.

3. Numerical results

The effectiveness of the proposed strategy is tested on a benchmark structure by considering heat conduction problems. The results are obtained using the NURBS-density-based algorithm developed in previous works [9, 13] and implemented in the in-house software SANTO. The software is coded in a Python® environment and it is interfaced with the FE code ANSYS ®. The penalisation schemes used in the following analysis are the SIMP [3] and the RAMP [14] schemes, which read :

$$\phi_{Ke}=\begin{cases}\rho_e^p & \text{for SIMP,}\\ \frac{\rho_e}{1+q(1-\rho_e)} & \text{for RAMP,}\end{cases} \tag{Eq. 9}$$

where p and q are the penalty parameters used for SIMP and RAMP schemes, respectively. In the following of this paper they have been set as $p=3$ and $q=8$. The benchmark chosen to perform compliance and volume, whose analytical expression analysis is shown in Fig. 1 and the geometrical parameters defining the design domain are : $a=20$ mm, $a_t=2$ mm and the thickness is $t=1$ mm. The FE model is made of $N_e=80\times80$ plane elements, with four nodes and one degree of freedom (the temperature) per node. The PPs of the thermal conductivity matrix of the constitutive lamina are $T=1$ $\mathrm{Wm^{-1}K^{-1}}$, $R=1$ $\mathrm{Wm^{-1}K^{-1}}$ and $\Phi=0$. The goal of this analysis is to investigate the influence of the penalty scheme used for the element stiffness matrix on the optimised configuration of the VSC

structure. The maximum number of iterations is set as $N_{\max}^{\text{iter}} = 800$, whilst the volume fraction is set as $\gamma = 0.3$. The CNLPP in Eq. 8 is solved by using a NURBS surface with $n_{\text{CP}} = 68 \times 68$ CPs and degrees of the Bernstein's polynomials equal to $p_1 = p_2 = 2$ for the pseudo-density field, and a B-Spline surface with the same integer parameters for the PPs field.

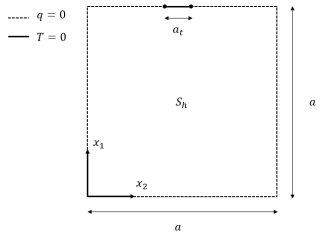
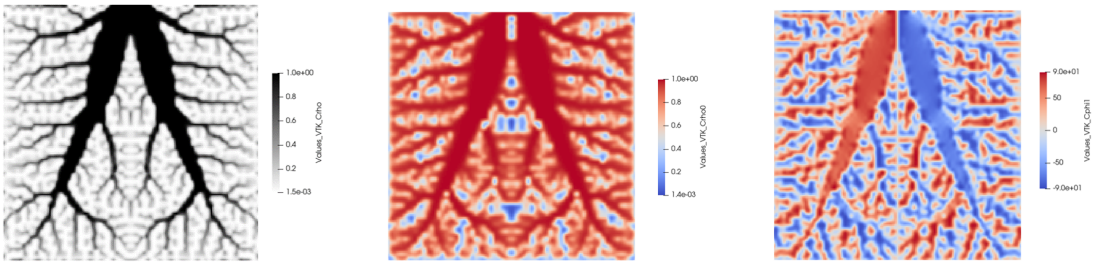


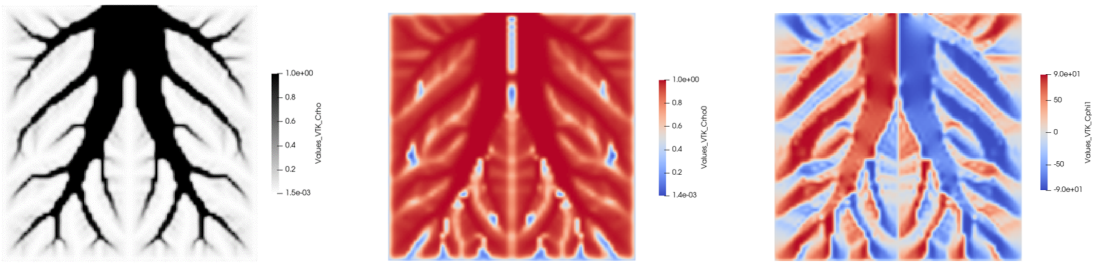
Fig. 1. – Geometry and boundary conditions of the benchmark structure taken from [8]

The optimised configurations of the VSC structure for the different penalty schemes are illustrated in Figs. 2-3 in terms of compliance and number of iterations N_{iter} to achieve convergence.
The following remarks can be drawn. Firstly, comparing the penalisation schemes, it is noteworthy the



(a) Topology field distribution (b) Polar parameter α field distribution (c) Polar parameter ϕ field distribution

Fig. 2. – Benchmark 1 : concurrent optimisation, SIMP scheme, $C = 0.1964$ Wmm, $N_{\text{iter}} = 800$



(a) Topology field distribution (b) Polar parameter α field distribution (c) Polar parameter ϕ field distribution

Fig. 3. – Benchmark 1 : concurrent optimisation, RAMP scheme, $C = 0.1566$ Wmm, $N_{\text{iter}} = 800$

difference between the size of the branches : for SIMP scheme thinner branches are obtained than the RAMP counterpart, additionally the number of branches is smaller for the latter. The lower thermal compliance is obtained using the RAMP scheme. As expected, the in-plane thermal conductivity matrix tends to exhibit an orthotropic behaviour ($\alpha = 1$) within the region corresponding to the solid phase, i.e., for those elements characterised by $\rho = 1.0$. Moreover, in such a region the main axis of orthotropy, related to the dimensionless PP ϕ, is aligned with the topological branches. More numerical results will be presented during the conference.

4. Conclusion

In this work, the concurrent optimisation of both the topology and anisotropy field descriptors is addressed for heat transfer problems. Specifically, the heat conduction problem has been reformulated and studied in the framework of the MS2L by considering NURBS entity to describe both the topology and anisotropy field. A study on the influence of the penalisation scheme on the optimised topology has been conducted, concluding that RAMP scheme leads to better performance than the SIMP conterpart in terms of thermal compliance. Further analyses are ongoing and will be shown during the conference.

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