Optimisation simultanée de la topologie et de l'anisotropie de composites à rigidité variable en prenant en compte des critères de résistance

Strength-based concurrent topology and anisotropy optimisation of variable-stiffness composite structures

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Résumé

Cet article présente un nouveau paradigme pour relever le défi de l'optimisation simultanée de la topologie et de l'anisotropie d'une structure composite à rigidité variable issue de la technologie de fabrication additive. L'approche proposée est basée sur une stratégie d'optimisation multi-échelle à deux niveaux, qui s'appuie sur des entités splines à base rationnelle non uniforme (NURBS) et sur le formalisme polaire. Les entités NURBS sont utilisées pour décrire à la fois la topologie et la distribution des paramètres polaires et de l'épaisseur sur la structure. Une formulation unifiée des critères de rupture au niveau du stratifié est présentée, et la notion de charge de rupture est introduite dans la formulation du problème afin d'obtenir des solutions optimisées qui ne dépendent pas de l'intensité des charges appliquées. Dans ce contexte, l'expression formelle du gradient des exigences de conception est dérivée en tirant parti des propriétés des entités NURBS. L'efficacité de l'approche est ensuite testée sur une série de problèmes de référence tirés de la littérature existante.

Abstract

This paper introduces a novel paradigm to address the challenge of concurrently optimising the topology and anisotropy of a variable-stiffness composite structures from additive manufacturing technology. The proposed approach is based on a multi-scale two-level optimisation strategy, which relies on non-uniform rational basis spline (NURBS) entities and the polar formalism. NURBS entities are used to describe both the topology and the distribution of the polar parameters and the thickness over the structure. A unified formulation of laminate-level failure criteria is presented, and the notion of failure load is introduced in the problem formulation to obtain optimised solutions that do not depend on the magnitude of the applied loads. In this context, the formal expression of the gradient of the design requirements is derived by taking advantage of the properties of the NURBS entities. The effectiveness of the approach is then tested on a series of benchmark problems taken from existing literature.

Mots Clés: Optimisation topologique, Anisotropie, NURBS, Critères de rupture, Fabrication additive
Keywords: Topology optimisation, Anisotropy, NURBS, Failure criteria, Additive manufacturing

1. Introduction

One of the most challenging problems in solid mechanics is the simultaneous optimisation of the topology and of the nature and orientation of the constitutive material of the continuum. Traditionally, this is achieved at two distinct levels of exchangeable order: structural design and material design.
The goal of structural design is to achieve the optimal design of a structure/system, taking into account the topology, boundaries and size of the structure. In the conceptual design stage, topology optimisation (TO) is a common practice, with the objective of identifying the most efficient distribution of a given material in the design domain, and the actual connectivity of its holes, resulting in an optimal performance of the structure. Over the past two decades, there has been significant theoretical and numerical development of TO methods. The most common approaches include density-based methods [1] and the level-set method [2].

In a different vein, anisotropic structures are frequently devised using multilayer composites with straight fibre format for constant stiffness, or curvilinear fibre format for variable stiffness, potentially with variable thickness and volume fraction. Research findings have demonstrated that the variable-stiffness design exhibits superior properties in a range of areas, including but not limited to strength [3], buckling load [4], stiffness [5], and combinations of these [6].

One of the most efficient methodologies to design VSC structures is the multi-scale multi-level design approach. In this context, the design problem is often split into two (or more) levels. Specifically, at the macroscopic scale the VSC laminate is modelled as an equivalent single-layer plate whose point-wise anisotropic behaviour is described through a convenient representation of its characteristic stiffness tensors. A possible way to represent anisotropy is through Verchery's polar formalism [7]. The polar formalism was initially developed for the classical laminate theory and later generalised to higher-order equivalent single-layer theories by Montemurro [8, 9]. The polar formalism allows expressing any plane tensor through invariants related to the symmetries of the tensor. The polar formalism has been employed in developing VSCs using the Multi-Scale Two-Level (MS2L) optimisation strategy as outlined by Montemurro et al. [3, 4, 5, 6].

Regarding the concurrent optimisation of topology and anisotropy of VSC structures, Ranaivomiarana et al. [10] considered the problem of the minimisation of the work of external forces of a structure subjected to homogeneous Dirichlet's boundary conditions (BCs) using a density-based TO algorithm, based on the solid isotropic material with penalisation (SIMP) scheme, to describe the density distribution of the material, and the polar parameters (PPs) to describe the anisotropic behaviour. To carry out the solution search, they used the optimality criterion of the alternate directions developed by Allaire et al. [11]. A similar approach has been recently proposed by Vertonghen et al. [12], where a gradient-based algorithm is used to solve in parallel three sub-problems for the optimisation of the pseudo-density field, the orientation angle of the main orthotropy axis and the anisotropic polar moduli. In [10, 12], a filter is applied only to the pseudo-density field to reduce the checker-board effect, but not to PPs fields, which are, hence, discontinuous, and the obtained solution are, consequently, non-manufacturable. Furthermore, the theoretical framework presented in [10, 12] lacks generality as it does not address the problem of the maximisation of the structural stiffness in presence of inhomogeneous Neumann-Dirichlet BCs and because it does not consider out-of-plane loads (thus the bending stiffness tensor of the VSC laminate is not optimised) and the possibility of also optimising the thickness distribution of the VSC laminate.

To overcome the above limitations, in [13] we proposed a general design strategy to face the problem of the concurrent design of topology and anisotropy of VSC structures in the framework of Non-Uniform Rational Basis Spline (NURBS) entities. Specifically, a general formulation of the optimisation problem based on the concept of generalised compliance has been presented by considering the general case of VSC structures having variable thickness subjected to inhomogeneous Neumann-Dirichlet BCs and both in-plane and out-of-plane loads. The basic idea behind the optimisation strategy based on NURBS entities is that each field of design variables of dimension D is modelled by a NURBS entity of dimension $D + 1$. Since the NURBS entity is a purely geometrical entity, the optimised solution does not depend on the mesh of the Finite Element (FE) model. Moreover, the continuity and differentiability of the NURBS entity can be tuned acting on different parameters (degrees of the basis functions, knot vectors components), and its main properties, such as the local support property, can be efficiently exploited to circumvent all the issues mentioned above (e.g., checker-board effect) [13].

To the best of the authors' knowledge, the problem of concurrently optimising the topology and anisotropy of VSC structures, whilst considering design requirements related to strength, has not been addressed in the literature. The aim of this work is to fill this gap by proposing an unified formulation of the main phenomenological criteria for anisotropic materials, i.e., Tsai–Wu, Hoffman and Tsai–Hill criteria, at the laminate level. The study also aims to address common issues related to stress-based TO problems, such as local behaviour and singularity of stresses, by introducing a special adaptive

χ-norm aggregation function that avoids overflow/underflow issues, and a modified pq relaxation approach. The effectiveness of the proposed approach is tested on meaningful benchmark problems taken from the literature.

2. Concurrent optimisation of the topology and anisotropy fields of a variable-stiffness composite structure

The design problem of a VSC structure in the framework of the MS2L optimisation strategy is split in two problems formulated at different levels/scales. In this work, only the first-level problem is addressed, which focuses on the VSC macroscopic scale. In this context, the goal is to search for the optimal value of the topology and PPs fields satisfying the design requirements of the problem at hand.

2.1. Topological field design variables

In the NURBS-density-based method [14], NURBS entities are used to represent the pseudo-density field. For a 2D problem a 3D NURBS surface is used, whose third coordinate is the pseudo-density field that reads:

$$\rho\left(\zeta_1, \zeta_2\right) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} R_{i,j}\left(\zeta_1, \zeta_2\right) \rho_{i,j}, \qquad \text{(Eq. 1)}$$

where $n_i + 1$ is the number of Control Points (CPs) along the i-th direction and $n_{\mathrm{CP}} = (n_1 + 1)(n_2 + 1)$ the number of CPs constituting the control net of the surface; $\rho_{i,j}$ is the value of the pseudo-density field at the generic CP, whilst the functions $R_{i,j}$ are the piece-wise rational basis functions of the NURBS entity [15], which depend on the weights $w_{i,j}$, and the Bernstein's polynomials N_{i,p_k}, as widely discussed in [15]. The dimensionless parameters ζ_j can be related to the spatial coordinates x_j as:

$$\zeta_j := \frac{x_j}{L_j},\ j = 1, 2, 3, \qquad \text{(Eq. 2)}$$

where L_j is the characteristic length of the domain along the x_j axis. In this background, the design variables are the pseudo-density $\rho_{i,j}$ of the generic CP and the associated weight $w_{i,j}$ that are collected in the following vector:

$$\boldsymbol{\xi}_1^{\mathrm{T}} = \left(\rho_{i,j}, w_{i,j}\right)_{\substack{1 \le i \le n_1 \\ 1 \le j \le n_2}},\ \boldsymbol{\xi}_1 \in \mathbb{R}^{2n_{\mathrm{CP}}}. \qquad \text{(Eq. 3)}$$

2.2. Anisotropy and thickness fields design variables

At the macroscopic scale, the VSC laminate can be modelled as an equivalent homogeneous anisotropic single-layer plate. If the VSC laminate is quasi-homogeneous and orthotropic [3], its local response, in terms of stiffness and strength, is described by its thickness t, and three mechanical variables, i.e., the PPs $R_{0K}^{A^*}$, $R_1^{A^*}$ and $\Phi_1^{A^*}$. Let t_{LB} and t_{UB} denote the lower and upper bounds of the thickness of the laminate, respectively, and let R_0, R_1, Φ_1 be the anisotropic moduli and the second polar angle of the reduced in-plane stiffness matrix $\mathbf{Q}_{\mathrm{in}}$. For optimisation purposes, the following dimensionless quantities are introduced:

$$\tau = \frac{t - t_{\mathrm{LB}}}{t_{\mathrm{UB}} - t_{\mathrm{LB}}}, \quad \rho_{0K} = \frac{R_{0K}^{A^*}}{R_0}, \quad \rho_1 = \frac{R_1^{A^*}}{R_1}, \quad \phi_1 = \frac{2\Phi_1^{A^*}}{\pi}. \qquad \text{(Eq. 4)}$$

It is noteworthy that the dimensionless anisotropic moduli ρ_{0K} and ρ_1 in (Eq. 4) must satisfy the point-wise geometrical feasibility conditions introduced by Vannucci [7] to guarantee they can be matched by, at least, one stacking sequence as a result of the second-level problem [5]. Izzi et al. [3]

introduced a change of variables to remap the feasible domain in the PPs space over the unit square $[0, 1] \times [0, 1]$ as follows:

$$(\alpha_0, \alpha_1) \coloneqq \left(\frac{\rho_{0K} - 1}{2(\rho_1^2 - 1)}, \rho_1 \right), \tag{Eq. 5}$$

Taking α_0 and α_1 as design variables instead of ρ_{0K} and ρ_1 makes the design problem easier, since the feasibility conditions on PPs [7] are implicitly satisfied. Therefore, the design variables fields describing the macroscopic behaviour of the VSC are $\tau\ (\zeta_1, \zeta_2)$, $\alpha_0\ (\zeta_1, \zeta_2)$, $\alpha_1\ (\zeta_1, \zeta_2)$ and $\phi_1\ (\zeta_1, \zeta_2)$. Contrary to the pseudo-density field ρ used for TO, only Basis Spline (B-spline) entities are used to describe the spatial variation of these variables, since no oscillations occur when optimising the PPs in the context of the MS2L optimisation strategy [5, 3, 4]. Therefore, the design variables are the dimensionless quantities $\tau,\ \alpha_0,\ \alpha_1,\ \phi_1$ computed at the CPs of the B-spline entity, which are collected in the following vector:

$$\boldsymbol{\xi}_2^{\mathrm{T}} = \left(\tau_{i,j}, \alpha_{0i,j}, \alpha_{1i,j}, \phi_{1i,j} \right)_{\substack{1 \le i \le n_1 \\ 1 \le j \le n_2}}, \quad \boldsymbol{\xi}_2 \in \mathbb{R}^{4n_{\mathrm{CP}}}. \tag{Eq. 6}$$

2.3. Design requirements

Two design requirements are included in the optimisation problem of the VSC structure: the failure load and the lightness. Specifically, the goal is to find the optimal distribution of the topological and anisotropy descriptors that maximises the failure load subject to a constraint on the volume.
The failure load is the minimum value of the factor to be applied to the external loads to observe failure in the the VSC structure. It is defined as:

$$\lambda_{\min} \coloneqq \min_{e \in \Omega_{\mathrm{DR}}} \lambda_e, \tag{Eq. 7}$$

where λ_e is the positive root of the following equation (which is computed for each element of the mesh of the FE model)

$$Q_e \lambda_e^2 + L_e \lambda_e - s_{\mathrm{th}} = 0, \tag{Eq. 8}$$

where $s_{\mathrm{th}} \le 1$ is a security factor to be set by the user, whereas the scalars Q_e and L_e read

$$Q_e \coloneqq \boldsymbol{\varepsilon}_{0,e}^{\mathrm{T}} \frac{\mathbf{G}_{\mathrm{lam},e}}{t_e} \boldsymbol{\varepsilon}_{0,e}\,, \quad L_e \coloneqq \boldsymbol{\varepsilon}_{0,e}^{\mathrm{T}} \frac{\mathbf{g}_{\mathrm{lam},e}}{t_e}. \tag{Eq. 9}$$

In (Eq. 9), $\boldsymbol{\varepsilon}_{0,e}\ (\rho, \alpha_0, \alpha_1, \phi_1, \tau)$ is the strain vector of the element (Voigt's notation) calculated when $\lambda_e = 1$ (reference structure), t_e is the local thickness of the VSC laminate, whilst $\mathbf{G}_{\mathrm{lam},e}\ (\rho, \alpha_0, \alpha_1, \phi_1, \tau)$ and $\mathbf{g}_{\mathrm{lam},e}\ (\rho, \alpha_0, \alpha_1, \phi_1, \tau)$ are the VSC laminate strength matrix and vector, respectively, whose expression is given in [3].
The lightness requirement is here expressed through a constraint on the volume of the structure:

$$g\ (\rho, \tau) \coloneqq \frac{V\ (\rho, \tau)}{V_{\mathrm{ref}}} - \gamma, \tag{Eq. 10}$$

where V_{ref} and γ are the reference value of the volume and the volume fraction set by the user, while the expression of the volume of the structure V is

$$V \coloneqq \sum_{e=1}^{N_e} \rho_e \int_{A_e} t dS \approx \sum_{e=1}^{N_e} \rho_e\, A_e\, t_e, \tag{Eq. 11}$$

where A_e is the area of the generic element and ρ_e is the pseudo-density field computed at the centroid of the element.

2.4. Problem formulation

The design problem of the VSC structure is formulated as a Constrained Non-Linear Programming Problem (CNLPP) as:

$$\begin{aligned} &\min_{(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)} \delta_{\mathrm{max}}\left(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2\right), \text{ with } \delta_{\mathrm{max}} = \lambda_{\mathrm{min}}^{-1}, \\ &\text{subject to :} \\ &\begin{cases} g\left(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2\right) \leq 0, \\ \boldsymbol{\xi}_1 \in \left[\rho_{\mathrm{LB}},\rho_{\mathrm{UB}}\right]^{n_{\mathrm{CP}}} \times \left[w_{\mathrm{LB}},w_{\mathrm{UB}}\right]^{n_{\mathrm{CP}}}, \\ \boldsymbol{\xi}_2 \in [0,1]^{n_{\mathrm{CP}}} \times [0,1]^{n_{\mathrm{CP}}} \times [0,1]^{n_{\mathrm{CP}}} \times [-1,1]^{n_{\mathrm{CP}}}. \end{cases} \end{aligned} \qquad \text{(Eq. 12)}$$

In (Eq. 12), ρ_{LB}, ρ_{UB}, w_{LB} and w_{UB} are user-defined bounds on the pseudo-density value and the associated weight at each CP. Problem (Eq. 12) has been solved using a gradient-based algorithm. The formal expression of the gradient of the structural responses δ_{max} and V will be shown during the conference.

3. Numerical results

The effectiveness of the proposed method is illustrated through a benchmark structure withstanding in-plane loads. More test cases will be presented during the speech. The results presented here are obtained by merging the codes SANTO (SIMP And NURBS for Topology Optimisation) [14] and VISION (VarIable Stiffness composItes Optimisation based on NURBS) [5] developed at the I2M laboratory in Bordeaux. Both software are coded in the Python® environment and are interfaced with the FE code ANSYS®, which is used to generate the FE model of the structure and to assess the structural responses.

The benchmark structure, whose geometry and BCs are shown in Fig. 1a, is modelled using 6400 four-node quadrilateral shell elements (ANSYS SHELL181) with six degrees of freedom per node. The element kinematics is based on the first-order shear deformation theory and a force $F = 5$ N is evenly distributed on five nodes as shown in Fig. 1a. In this case, the thickness is not included among the design variables and is considered as uniform across the design domain and equal to $t = 1$ mm. The elastic and strength properties of the constitutive lamina are: $E_1 = 142000$ MPa, $E_2 = 103000$ MPa, $G_{12} = 7200$ MPa, $\nu_{12} = 0.27$, $\nu_{23} = 0.54$, $X_t = 2280$ MPa, $X_c = 1440$ MPa, $Y_t = Z_t = 57$ MPa, $Y_c = Z_c = 228$ MPa, $S_{xy} = S_{xz} = 71$ MPa, $S_{yz} = 40$ MPa. The PPs are initialised as uniform fields as $\rho_{0K} = 0$, $\rho_1 = 0$, $\phi_1 = 0$, whilst the pseudo-density field is initialised in order to fulfil the constraint on the volume fraction of Eq. (Eq. 10), which has been set as $\gamma = 0.4$. For all the analyses, problem (Eq. 12) is solved by using a NURBS surface with $n_{\mathrm{CP}} = 85 \times 85$ CPs and degrees of the Bernstein's polynomials equal to $p_1 = p_2 = 2$ for the pseudo-density field, and a B-spline surface with the same integer parameters is used to describe each PP field.

The optimised configuration of the VSC structure is illustrated in Fig. 1 in terms of optimised topology (Fig. 1b), and optimised distributions of the PPs over the domain (Figs. 1c-1e). As expected, the optimised solution exhibit a connecting radius in the region where stress concentration occurs (see Fig. 1b). The VSC composite exhibits a standard orthotropic behaviour in the horizontal and some diagonal branches (red colour in Figs. 1c and 1d), whereas it exhibits a square symmetric behaviour in the rest of the design domain (blue colour in Fig. 1d). As expected, the main orientation of the orthotropy axis aligns with the local stress field (see Fig. 1e). This solution exhibits a load factor $\lambda_{\mathrm{min}} = 1111,73$ that correspond to a maximum external force $F = 5558.64$ N that can be applied without causing failure in the VSC structure.

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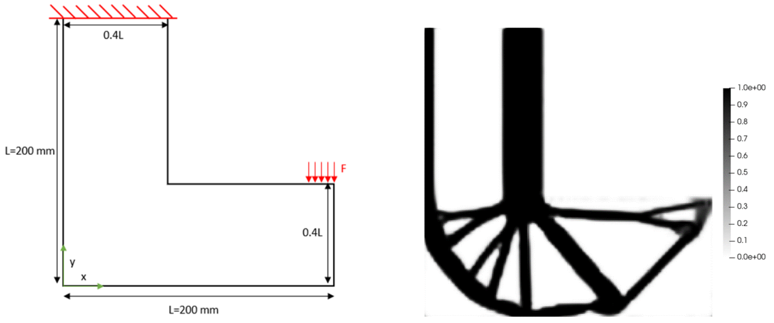
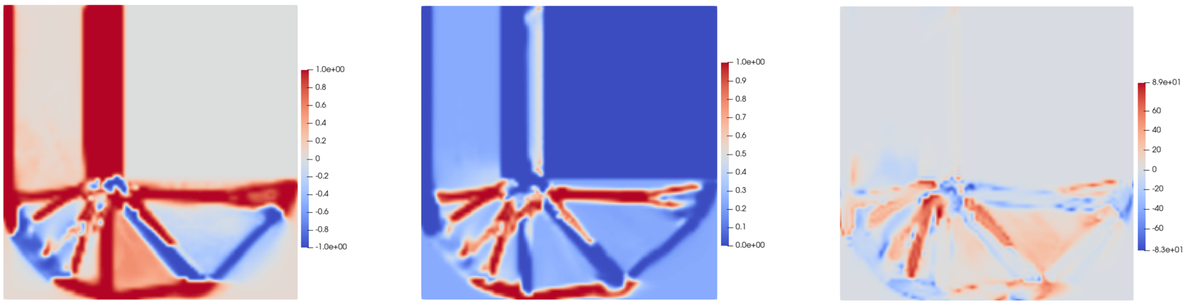


(a) Geometry and BCs (b) ρ



(c) ρ_{0K} (d) ρ_1 (e) ϕ_1

Fig. 1.: Optimised configuration of the VSC structure for problem (Eq. 12)

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